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2. (Heat and Thermodynamics)

• First law of Thermodynamics —

The amount of heat given to a system is equal to the sum of the increase in the internal energy of the system and the external work done.

$$\bullet dQ = dU + dW$$

Where,

$dQ \rightarrow$ amount of heat supplied

$dU \rightarrow$ increase in internal energy

$dW \rightarrow$ amount of external work done

Also, ~~according~~ according to joule,
The quantity of heat produced is directly proportional to the mechanical work done.

$$W \propto H$$

or,

$$\therefore W = JH$$

Here,

$W \rightarrow$ Work done.

$H \rightarrow$ produced Heat.

$J \rightarrow$ constant.

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- Isothermal Process - An isothermal process is a thermodynamic process in which the temperature (T) of a gas remains constant throughout the process. ($\Delta T = 0$)

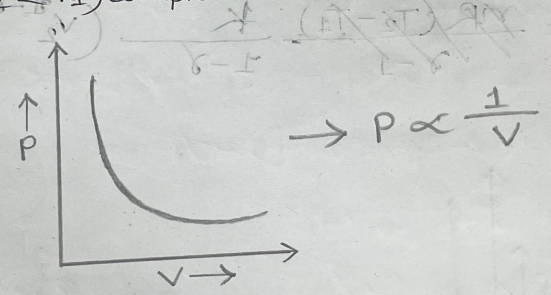
Relation:

$PV = \text{constant}$
 or,
 $P_1 V_1 = P_2 V_2$

$W = nRT \ln \left(\frac{V_2}{V_1} \right)$

if ($V_2 > V_1$) expansion $\rightarrow$ work done by the gas (P)
 if ($V_2 < V_1$) compression $\rightarrow$ work done on the gas (N)

Graph:



- Adiabatic Process - An adiabatic process is a thermodynamic process in which no heat is exchanged between the system and its surroundings. ($Q = 0$)

Relation:

$PV^\gamma = \text{constant}$

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or $V^{\gamma-1} = \text{constant}$

$\frac{P^{1-\gamma}}{T} = \text{constant}$

• For monoatomic (अणुआणु गैस) gas [He, Ar] $\rightarrow$
 $\gamma = 1.67$ ($i=3$)

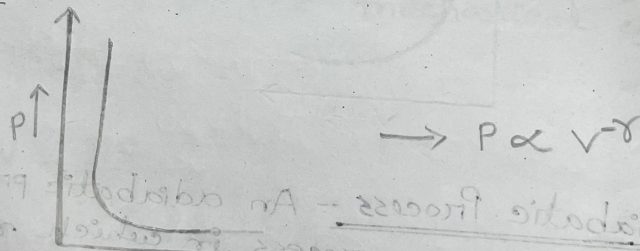
• For diatomic (द्विअणु गैस) gas [N₂, O₂] $\rightarrow$
 $\gamma = 1.4$ ($i=5$)

• अणुआणु गैस $\rightarrow \gamma = 1.33$ ($i=6$)
 (polyatomic)

$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$

or,
 $W = \frac{\gamma P (T_2 - T_1)}{\gamma - 1} = \frac{\gamma}{\gamma - 1} (P_2 V_2^{1-\gamma} - P_1 V_1^{1-\gamma})$

Graph:



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• Application of first law of Thermodynamics —

We consider a system of volume V , Pressure P , Temperature T , internal energy U containing 1 gm mole ideal gas.

If dW is the external work done then applying first law of thermodynamics —

$$dH = du + dW$$

$$\Rightarrow dH = du + PdV \quad [\because dW = PdV]$$

For adiabatic process,

$$dH = 0$$

$$\Rightarrow du + PdV = 0$$

$$\therefore C_V = \frac{du}{dT} \quad [m=1]$$

$$\Rightarrow C_V = \frac{du}{dT}$$

$$\Rightarrow du = C_V dT$$

$$\therefore C_V dT + PdV = 0 \quad \text{--- (1)}$$

We know from ideal gas equation,

$$PV = nRT \quad [n=1]$$

$$\Rightarrow PV = RT$$

$$\Rightarrow P = \frac{RT}{V}$$

differentiating,

$$Pdv + Vdp = RdT$$

$$\Rightarrow dT = \frac{Pdv + Vdp}{R}$$

Putting this value on equation (1) and we get,

$$C_v \left(\frac{Pdv + Vdp}{R} \right) + Pdv = 0$$

$$\Rightarrow \frac{C_v Pdv + C_v Vdp}{R} + Pdv = 0$$

$$\Rightarrow C_v Pdv + C_v Vdp + R Pdv = 0$$

$$\Rightarrow C_v Pdv + C_v Vdp + (C_p - C_v) Pdv = 0 \quad [\because R = (C_p - C_v)]$$

$$\Rightarrow Pdv (C_v + C_p - C_v) + C_v Vdp = 0$$

$$\Rightarrow C_p Pdv + C_v Vdp = 0$$

$$\Rightarrow \frac{C_p Pdv}{C_v} + \frac{C_v Vdp}{C_v} = 0 \quad [\because \text{dividing with } C_v]$$

$$\Rightarrow \gamma Pdv + Vdp = 0 \quad [\because \gamma = \frac{C_p}{C_v}]$$

$$\Rightarrow \frac{\gamma Pdv}{P} + \frac{Vdp}{P} = 0 \quad [\because \text{dividing with } P]$$

$$\Rightarrow \frac{1}{V} \cdot \gamma dv + \frac{1}{P} dp = 0$$

by integrating,

$$\int \frac{1}{p} \cdot dp + \int \gamma \frac{1}{v} \cdot dv = 0$$

$$\Rightarrow \ln p + \gamma \ln v = \ln c \text{ (const)} \quad [\because \int \frac{1}{x} \cdot dx = \ln x]$$

$$\Rightarrow \ln p + \ln v^\gamma = \ln c$$

$$\Rightarrow \ln(pv^\gamma) = \ln c \quad [\because \ln a + \ln b = \ln(ab)]$$

$$\Rightarrow \boxed{pv^\gamma = \text{constant}} \quad \text{----- (2)}$$

again,

$$pv = nRT$$

$$\Rightarrow pv = RT \quad [\because n=1]$$

$$\Rightarrow p = \frac{RT}{v}$$

Putting this value on equation (2) and get,

$$\frac{RT}{v} v^\gamma = c$$

$$\Rightarrow RT \cdot \frac{v^\gamma}{v} = c$$

$$\Rightarrow RT \cdot v^{\gamma-1} = c$$

$$\Rightarrow \boxed{T v^{\gamma-1} = \text{constant}}$$

and,

$$v = \frac{RT}{p}$$

$$\Rightarrow p \cdot \left(\frac{RT}{p}\right)^\gamma = c$$

$$\Rightarrow p \cdot \frac{R^\gamma T^\gamma}{p^\gamma} = c$$

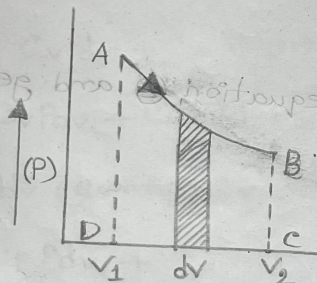
$$\Rightarrow \frac{P}{P^\gamma} \cdot R^\gamma T^\gamma = c$$

$$\Rightarrow P^{1-\gamma} \cdot R^\gamma T^\gamma = c$$

$$\Rightarrow P^{1-\gamma} \cdot T^\gamma = c$$

$$\Rightarrow T P^{\frac{1-\gamma}{\gamma}} = \text{constant}$$

• Work done by an Isothermal Process —



We consider a system of volume V , Pressure P and temperature T containing an ideal gas. If the gas is allowed to expand in an isothermal process then the work

is done by the gas. no pd work done.

Let the initial and final volumes be V_1 and V_2 respectively.

Therefore, work done will be the area of ABCD.

$$\therefore \text{Work done} = \int_{V_1}^{V_2} dw = \text{area ABCD}$$

$$= \int_{V_1}^{V_2} P \cdot dV \quad [\because dw = PdV]$$

$$= \int_{V_1}^{V_2} \frac{RT}{V} dV \quad [\because PV = RT \Rightarrow P = \frac{RT}{V}]$$

$$= RT \int_{V_1}^{V_2} \frac{1}{V} dV$$

$$= RT [\ln V]_{V_1}^{V_2}$$

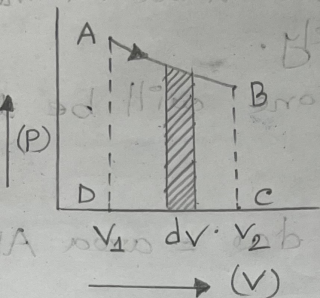
$$= RT [\ln V_2 - \ln V_1]$$

$$= RT \ln \left(\frac{V_2}{V_1} \right)$$

$$\therefore W = RT \ln \left(\frac{V_2}{V_1} \right)$$

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• Work done by an Adiabatic Process —



We consider a system of volume V , pressure P and temperature T containing an ideal gas. If the gas is allowed to expand in an adiabatic process then the work is done by the gas.

Let the initial and final volumes be V_1 and V_2 respectively.

Therefore work done will be the area of ABCD

$$\therefore \text{Work done} = \int_{V_1}^{V_2} dw = \text{area of ABCD}$$

$$= \int_{V_1}^{V_2} P \cdot dV \quad [\because dw = P \cdot dV]$$

During an adiabatic process,

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$$p v^\gamma = \text{const} (K)$$

$$\Rightarrow p = \frac{K}{v^\gamma}$$

$$\therefore W = \int_{v_1}^{v_2} \frac{K}{v^\gamma} dv$$

$$= K \int_{v_1}^{v_2} \frac{1}{v^\gamma} dv$$

$$= K \int_{v_1}^{v_2} v^{-\gamma} dv$$

$$= K \left[\frac{v^{-\gamma+1}}{-\gamma+1} \right]_{v_1}^{v_2}$$

$$= \frac{K}{1-\gamma} \left(v_2^{1-\gamma} - v_1^{1-\gamma} \right)$$

$$= \frac{K}{1-\gamma} \left(v_2^{-(\gamma-1)} - v_1^{-(\gamma-1)} \right)$$

$$= \frac{K}{1-\gamma} \left(\frac{1}{v_2^{\gamma-1}} - \frac{1}{v_1^{\gamma-1}} \right)$$

$$\therefore W = \frac{K}{1-\gamma} \left(\frac{1}{v_2^{\gamma-1}} - \frac{1}{v_1^{\gamma-1}} \right)$$

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• Second Law of Thermodynamics —

A reservoir is a device having infinite thermal capacity and which can absorb, retain or reject unlimited quantity of heat without any change in its temperature.

First Part: According to Kelvin,

"It is impossible to get a continuous supply of work from a body by cooling it to a temperature lower than that of its surroundings."

Second Part: According to Clausius,

"It is impossible to make heat flow from a body at a lower temperature to a body at a higher temperature without doing external work on the working substance."

$$\left(\frac{Q_1}{T_1} - \frac{Q_2}{T_2} \right) \frac{K}{T_1 - T_2} =$$

$$\left(\frac{Q_1}{T_1} - \frac{Q_2}{T_2} \right) \frac{K}{T_1 - T_2} = W \therefore$$

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Carnot's Engine

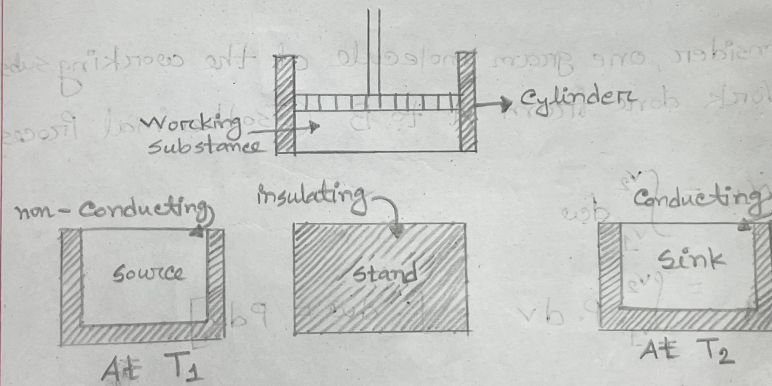
A Carnot engine is a reversible heat engine that operates between two heat reservoirs — a hot source at temperature T_1 and a cold sink at temperature T_2 .

$$\eta = 1 - \frac{T_2}{T_1}$$

where,

η = efficiency of the Carnot engine.

For any engine, there are three essential requisites:

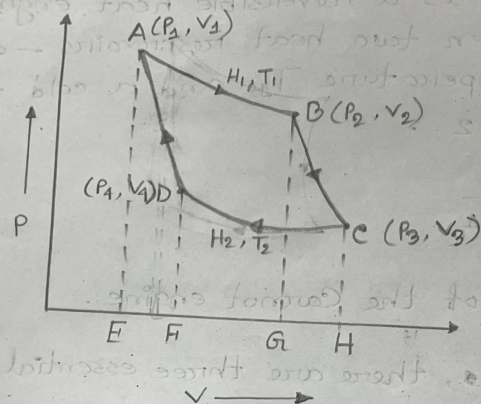


- (1) Source
- (2) Sink
- (3) Working substance.

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• Carnot's Cycle - (work done)



Consider one gram molecule of the working substance
 Work done from A to B (Isothermal Process)

$$W_1 = \int_{V_1}^{V_2} dw$$

$$= \int_{V_1}^{V_2} P \cdot dV \quad [\because dw = P dV]$$

$$= \int_{V_1}^{V_2} \frac{RT_1}{V} \cdot dV \quad \left[\because PV = nRT \Rightarrow P = \frac{RT}{V}, n=1 \right]$$

$$= RT_1 [\ln V]_{V_1}^{V_2}$$

$$= RT_1 (\ln V_2 - \ln V_1)$$

$$= RT_1 \ln \frac{V_2}{V_1} = \text{area of ABGE} \dots \text{--- (1)}$$

Work done from B to C (Adiabatic Process)

$$\begin{aligned}
 W_2 &= \int_{V_2}^{V_3} P \cdot dV \\
 &= \int_{V_2}^{V_3} \frac{K}{V^\gamma} dV \quad \left[\because PV^\gamma = K \Rightarrow P = \frac{K}{V^\gamma} \right] \\
 &= K \left[\frac{V^{1-\gamma}}{1-\gamma} \right]_{V_2}^{V_3} \\
 &= \frac{K}{1-\gamma} (V_3^{1-\gamma} - V_2^{1-\gamma}) \\
 &= \frac{K V_3^{1-\gamma} - K V_2^{1-\gamma}}{1-\gamma} \\
 &= \frac{P_3 V_3 - P_2 V_2}{1-\gamma} \\
 &= \frac{RT_3 - RT_2}{1-\gamma} \quad \left[\because PV = RT, n=1 \right] \\
 &= \frac{-R(T_3 - T_2)}{-(\gamma-1)} \\
 &= \frac{R(T_1 - T_2)}{\gamma-1} \\
 &= \text{area of BCHG} \quad \text{--- (2)}
 \end{aligned}$$

Work done from C to D (Isothermal Process)

$$W_3 = \int_{V_3}^{V_4} P \cdot dV$$

(1)

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$$\begin{aligned}
 &= RT_2 \ln \frac{V_4}{V_3} \\
 &= -RT_2 \ln \frac{V_3}{V_4} \\
 &= \text{area of CHFD} \text{ --- (3)}
 \end{aligned}$$

Work done from D to A (Adiabatic Process)

$$\begin{aligned}
 W_4 &= \int_{V_4}^{V_1} P \cdot dV \\
 &= - \frac{R(T_1 - T_2)}{\gamma - 1} \\
 &= \text{area of DFHA}
 \end{aligned}$$

$$\begin{aligned}
 \therefore W &= W_1 + W_2 + W_3 + W_4 \\
 &= RT_1 \ln \frac{V_2}{V_1} + \frac{R(T_1 - T_2)}{\gamma - 1} - RT_2 \ln \frac{V_3}{V_4} - \frac{R(T_1 - T_2)}{\gamma - 1} \\
 &= RT_1 \ln \frac{V_2}{V_1} - RT_2 \ln \frac{V_3}{V_4} \text{ --- (4)}
 \end{aligned}$$

The points A and D are on the same adiabatic.

$$T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\begin{aligned}
 \frac{T_1}{T_2} &= \left(\frac{V_4}{V_1} \right)^{\gamma-1} \\
 \Rightarrow \frac{T_2}{T_1} &= \left(\frac{V_1}{V_4} \right)^{\gamma-1} \text{ --- (5)}
 \end{aligned}$$

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The points B and C are on the same adiabatic

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V_3}{V_2} \right)^{\gamma-1}$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{V_2}{V_3} \right)^{\gamma-1} \quad \dots \dots \textcircled{6}$$

From $\textcircled{5}$ and $\textcircled{6} \Rightarrow$

$$\left(\frac{V_1}{V_4} \right)^{\gamma-1} = \left(\frac{V_2}{V_3} \right)^{\gamma-1}$$

$$\Rightarrow \frac{V_1}{V_4} = \frac{V_2}{V_3}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{V_4}{V_3}$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

From equation $\textcircled{4}$,

$$W = R T_1 \ln \frac{V_2}{V_1} - R T_2 \ln \frac{V_2}{V_1} = H_1 - H_2$$

$$= R \ln \frac{V_2}{V_1} [T_1 - T_2]$$

$$= R \ln \frac{V_2}{V_1} \cdot T_1 - R \ln \frac{V_2}{V_1} \cdot T_2$$

$$\therefore W = H_1 - H_2$$

$$\therefore \eta = \frac{W}{H_1} = \frac{H_1 - H_2}{H_1}$$

$$= \frac{H_1 - H_2}{H_1}$$

$$= \frac{R \ln \frac{V_2}{V_1} [T_1 - T_2]}{R \ln \frac{V_2}{V_1} T_1}$$

$$= \frac{T_1 - T_2}{T_1}$$

Here,

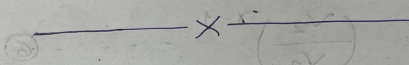
$$H_1 = R T_1 \ln \left(\frac{V_2}{V_1} \right)$$

$$H_2 = R T_2 \ln \left(\frac{V_2}{V_1} \right)$$

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$$= 1 - \frac{T_2}{T_1}$$

This is the efficiency of carnot cycle.



• Maxwell's Thermodynamics Relation -

We consider a system which pressure is P , volume is V , temperature T , entropy S and internal energy U .

The state of the system can be specified by any two of pair of these quantities taking independent variable.

According to 1st law of thermodynamics,

$$\partial H = du + \partial w$$

Here,

∂ = Partial change

d = Full change

or,

$$du = \partial H - Pdv$$

or,

$$du = Tds - Pdv$$

$$[\because ds = \frac{\partial H}{T}]$$

①

Let,

U, S, V are the function of any two

variables x and y , $f(x, y)$ where x and y maybe any two pair of the quantities. (P, V, T, S)

$$\therefore du = \left(\frac{\partial u}{\partial x}\right)_y dx + \left(\frac{\partial u}{\partial y}\right)_x dy$$

$$\therefore ds = \left(\frac{\partial s}{\partial x}\right)_y dx + \left(\frac{\partial s}{\partial y}\right)_x dy$$

$$\therefore dv = \left(\frac{\partial v}{\partial x}\right)_y dx + \left(\frac{\partial v}{\partial y}\right)_x dy$$

putting this values in equation (1) $\Rightarrow$

$$\begin{aligned} \left(\frac{\partial u}{\partial x}\right)_y dx + \left(\frac{\partial u}{\partial y}\right)_x dy &= T \left[\left(\frac{\partial s}{\partial x}\right)_y dx + \left(\frac{\partial s}{\partial y}\right)_x dy \right] \\ &\quad - P \left[\left(\frac{\partial v}{\partial x}\right)_y dx + \left(\frac{\partial v}{\partial y}\right)_x dy \right] \\ \Rightarrow \left(\frac{\partial u}{\partial x}\right)_y dx + \left(\frac{\partial u}{\partial y}\right)_x dy &= \left[T \left(\frac{\partial s}{\partial x}\right)_y - P \left(\frac{\partial v}{\partial x}\right)_y \right] dx \\ &\quad + \left[T \left(\frac{\partial s}{\partial y}\right)_x - P \left(\frac{\partial v}{\partial y}\right)_x \right] dy \end{aligned}$$

Comparing the co-efficient of dx and dy ,

$$\left(\frac{\partial u}{\partial x}\right)_y = T \left(\frac{\partial s}{\partial x}\right)_y - P \left(\frac{\partial v}{\partial x}\right)_y \quad \text{--- (2)}$$

$$\left(\frac{\partial u}{\partial y}\right)_x = T \left(\frac{\partial s}{\partial y}\right)_x - P \left(\frac{\partial v}{\partial y}\right)_x \quad \text{--- (3)}$$

Differentiating equation (2) with respect to y

and ③ with respect to x ,

$$\frac{\partial^2 u}{\partial y \partial x} = T \left(\frac{\partial^2 s}{\partial x \partial y} \right) + \left(\frac{\partial s}{\partial x} \right) y \cdot \left(\frac{\partial T}{\partial y} \right) x - p \left(\frac{\partial^2 v}{\partial x \partial y} \right) - \left(\frac{\partial v}{\partial x} \right) y \cdot \left(\frac{\partial p}{\partial y} \right) x \quad \text{--- ④}$$

$$\frac{\partial^2 u}{\partial x \partial y} = T \left(\frac{\partial^2 s}{\partial y \partial x} \right) + \left(\frac{\partial s}{\partial y} \right) x \cdot \left(\frac{\partial T}{\partial x} \right) y - p \left(\frac{\partial^2 v}{\partial y \partial x} \right) - \left(\frac{\partial v}{\partial y} \right) x \cdot \left(\frac{\partial p}{\partial x} \right) y \quad \text{--- ⑤}$$

Since, changing internal energy du is due to changing dv and temperature dT , Therefore, du is perfect differential.

$$\begin{aligned} \therefore \frac{\partial^2 u}{\partial y \partial x} &= \frac{\partial^2 u}{\partial x \partial y} \\ \Rightarrow T \left(\frac{\partial^2 s}{\partial y \partial x} \right) + \left(\frac{\partial s}{\partial x} \right) y \cdot \left(\frac{\partial T}{\partial y} \right) x - p \left(\frac{\partial^2 v}{\partial y \partial x} \right) - \left(\frac{\partial v}{\partial y} \right) x \cdot \left(\frac{\partial p}{\partial x} \right) y \\ &= T \left(\frac{\partial^2 s}{\partial x \partial y} \right) + \left(\frac{\partial s}{\partial y} \right) x \cdot \left(\frac{\partial T}{\partial x} \right) y - p \left(\frac{\partial^2 v}{\partial x \partial y} \right) - \left(\frac{\partial v}{\partial x} \right) y \cdot \left(\frac{\partial p}{\partial y} \right) x \\ \Rightarrow \left(\frac{\partial s}{\partial x} \right) y \cdot \left(\frac{\partial T}{\partial y} \right) x - \left(\frac{\partial v}{\partial x} \right) y \cdot \left(\frac{\partial p}{\partial y} \right) x &= \left(\frac{\partial s}{\partial y} \right) x \cdot \left(\frac{\partial T}{\partial x} \right) y - \left(\frac{\partial v}{\partial y} \right) x \cdot \left(\frac{\partial p}{\partial x} \right) y \end{aligned}$$

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Question-

Prove that,

$$(i) \left(\frac{\partial S}{\partial V} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V$$

$$(ii) \left(\frac{\partial T}{\partial V} \right)_S = -T \left(\frac{\partial P}{\partial H} \right)_V$$

$$(iii) \left(\frac{\partial P}{\partial T} \right)_S \left(\frac{\partial V}{\partial S} \right)_T - \left(\frac{\partial P}{\partial S} \right)_T \left(\frac{\partial V}{\partial T} \right)_S = 1$$

$$(iv) \left(\frac{\partial H}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V$$

$$(v) \left(\frac{\partial H}{\partial P} \right)_T = -T \left(\frac{\partial V}{\partial T} \right)_P$$

$$(vi) \left(\frac{\partial T}{\partial P} \right)_S = T \left(\frac{\partial V}{\partial H} \right)_P$$

$$(vii) \left(\frac{\partial T}{\partial P} \right)_V \left(\frac{\partial S}{\partial V} \right)_P - \left(\frac{\partial T}{\partial V} \right)_P \left(\frac{\partial S}{\partial P} \right)_V = 1$$

Answer -(i) Let $x = x$, $v = y$

$$\left(\frac{\partial T}{\partial x}\right)_y = 1, \quad \left(\frac{\partial v}{\partial x}\right)_y = 1$$

$$\left(\frac{\partial T}{\partial y}\right)_x = 0, \quad \left(\frac{\partial v}{\partial y}\right)_x = 0$$

putting these values in equation (6) $\Rightarrow$

$$\left(\frac{\partial s}{\partial x}\right)_y \cdot 0 - 0 \cdot \left(\frac{\partial p}{\partial y}\right)_x = \left(\frac{\partial s}{\partial y}\right)_x \cdot 1 - 1 \cdot \left(\frac{\partial p}{\partial x}\right)_y$$

$$\Rightarrow \left(\frac{\partial s}{\partial y}\right)_x - \left(\frac{\partial p}{\partial x}\right)_y = 0$$

$$\Rightarrow \left(\frac{\partial s}{\partial y}\right)_x = \left(\frac{\partial p}{\partial x}\right)_y$$

$$\Rightarrow \left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

[Proved]

(ii) Let $s = x$, $v = y$

$$\left(\frac{\partial s}{\partial x}\right)_y = 1, \quad \left(\frac{\partial v}{\partial y}\right)_x = 1$$

$$\left(\frac{\partial s}{\partial y}\right)_x = 0, \quad \left(\frac{\partial v}{\partial x}\right)_y = 0$$

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putting this values in equation (6) $\Rightarrow$

$$1 \cdot \left(\frac{\partial T}{\partial y}\right)_x - 0 \cdot \left(\frac{\partial P}{\partial y}\right)_x = 0 \cdot \left(\frac{\partial T}{\partial x}\right)_y - 1 \cdot \left(\frac{\partial P}{\partial x}\right)_y$$

$$\Rightarrow \left(\frac{\partial T}{\partial y}\right)_x = - \left(\frac{\partial P}{\partial x}\right)_y$$

$$\Rightarrow \left(\frac{\partial T}{\partial v}\right)_s = - \left(\frac{\partial P}{\partial s}\right)_v$$

$$\Rightarrow \left(\frac{\partial T}{\partial v}\right)_s = - \left(\frac{\partial P}{\partial H}\right)_v \quad \left[\because \partial s = \frac{\partial H}{T} \right]$$

$$\Rightarrow \left(\frac{\partial T}{\partial v}\right)_s = - T \left(\frac{\partial P}{\partial H}\right)_v$$

[Proved]

(iii) Let $s = x$, $T = y$

$$\left(\frac{\partial s}{\partial x}\right)_y = 1 \quad , \quad \left(\frac{\partial T}{\partial y}\right)_x = 1$$

$$\left(\frac{\partial s}{\partial y}\right)_x = 0 \quad , \quad \left(\frac{\partial T}{\partial x}\right)_y = 0$$

putting this values in equation (6) $\Rightarrow$

$$1 \cdot 1 - \left(\frac{\partial v}{\partial x}\right)_y \cdot \left(\frac{\partial P}{\partial y}\right)_x = 0 \cdot 0 - \left(\frac{\partial v}{\partial y}\right)_x \cdot \left(\frac{\partial P}{\partial x}\right)_y$$

$$\Rightarrow \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial P}{\partial x}\right)_y - \left(\frac{\partial v}{\partial x}\right)_y \left(\frac{\partial P}{\partial y}\right)_x = -1$$

$$\Rightarrow \left(\frac{\partial v}{\partial x}\right)_y \left(\frac{\partial P}{\partial y}\right)_x - \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial P}{\partial x}\right)_y = 1$$

$$\Rightarrow \left(\frac{\partial v}{\partial s}\right)_T \left(\frac{\partial P}{\partial T}\right)_s - \left(\frac{\partial v}{\partial T}\right)_s \left(\frac{\partial P}{\partial s}\right)_T = 1 \quad [\text{Proved}]$$